5.1 Exercises

The *HM mathSpace®* CD-ROM and *Eduspace®* for this text contain step-by-step solutions to all odd-numbered exercises. They also provide Tutorial Exercises for additional help.

VOCABULARY CHECK: Fill in the blank to complete the trigonometric identity.

1. $\dfrac{\sin u}{\cos u} =$ ________
2. $\dfrac{1}{\sec u} =$ ________
3. $\dfrac{1}{\tan u} =$ ________
4. $\dfrac{1}{\sin u} =$ ________
5. $1 +$ ________ $= \csc^2 u$
6. $1 + \tan^2 u =$ ________
7. $\sin\left(\dfrac{\pi}{2} - u\right) =$ ________
8. $\sec\left(\dfrac{\pi}{2} - u\right) =$ ________
9. $\cos(-u) =$ ________
10. $\tan(-u) =$ ________

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–14, use the given values to evaluate (if possible) all six trigonometric functions.

1. $\sin x = \dfrac{\sqrt{3}}{2}, \quad \cos x = -\dfrac{1}{2}$
2. $\tan x = \dfrac{\sqrt{3}}{3}, \quad \cos x = -\dfrac{\sqrt{3}}{2}$
3. $\sec \theta = \sqrt{2}, \quad \sin \theta = -\dfrac{\sqrt{2}}{2}$
4. $\csc \theta = \frac{5}{3}, \quad \tan \theta = \frac{3}{4}$
5. $\tan x = \frac{5}{12}, \quad \sec x = -\frac{13}{12}$
6. $\cot \phi = -3, \quad \sin \phi = \dfrac{\sqrt{10}}{10}$
7. $\sec \phi = \dfrac{3}{2}, \quad \csc \phi = -\dfrac{3\sqrt{5}}{5}$
8. $\cos\left(\dfrac{\pi}{2} - x\right) = \dfrac{3}{5}, \quad \cos x = \dfrac{4}{5}$
9. $\sin(-x) = -\dfrac{1}{3}, \quad \tan x = -\dfrac{\sqrt{2}}{4}$
10. $\sec x = 4, \quad \sin x > 0$
11. $\tan \theta = 2, \quad \sin \theta < 0$
12. $\csc \theta = -5, \quad \cos \theta < 0$
13. $\sin \theta = -1, \quad \cot \theta = 0$
14. $\tan \theta$ is undefined, $\quad \sin \theta > 0$

In Exercises 15–20, match the trigonometric expression with one of the following.

(a) $\sec x$ (b) -1 (c) $\cot x$
(d) 1 (e) $-\tan x$ (f) $\sin x$

15. $\sec x \cos x$
16. $\tan x \csc x$
17. $\cot^2 x - \csc^2 x$
18. $(1 - \cos^2 x)(\csc x)$
19. $\dfrac{\sin(-x)}{\cos(-x)}$
20. $\dfrac{\sin[(\pi/2) - x]}{\cos[(\pi/2) - x]}$

In Exercises 21–26, match the trigonometric expression with one of the following.

(a) $\csc x$ (b) $\tan x$ (c) $\sin^2 x$
(d) $\sin x \tan x$ (e) $\sec^2 x$ (f) $\sec^2 x + \tan^2 x$

21. $\sin x \sec x$
22. $\cos^2 x(\sec^2 x - 1)$
23. $\sec^4 x - \tan^4 x$
24. $\cot x \sec x$
25. $\dfrac{\sec^2 x - 1}{\sin^2 x}$
26. $\dfrac{\cos^2[(\pi/2) - x]}{\cos x}$

In Exercises 27–44, use the fundamental identities to simplify the expression. There is more than one correct form of each answer.

27. $\cot \theta \sec \theta$
28. $\cos \beta \tan \beta$
29. $\sin \phi(\csc \phi - \sin \phi)$
30. $\sec^2 x(1 - \sin^2 x)$
31. $\dfrac{\cot x}{\csc x}$
32. $\dfrac{\csc \theta}{\sec \theta}$
33. $\dfrac{1 - \sin^2 x}{\csc^2 x - 1}$
34. $\dfrac{1}{\tan^2 x + 1}$
35. $\sec \alpha \cdot \dfrac{\sin \alpha}{\tan \alpha}$
36. $\dfrac{\tan^2 \theta}{\sec^2 \theta}$
37. $\cos\left(\dfrac{\pi}{2} - x\right)\sec x$
38. $\cot\left(\dfrac{\pi}{2} - x\right)\cos x$
39. $\dfrac{\cos^2 y}{1 - \sin y}$
40. $\cos t(1 + \tan^2 t)$
41. $\sin \beta \tan \beta + \cos \beta$
42. $\csc \phi \tan \phi + \sec \phi$
43. $\cot u \sin u + \tan u \cos u$
44. $\sin \theta \sec \theta + \cos \theta \csc \theta$

In Exercises 45–56, factor the expression and use the fundamental identities to simplify. There is more than one correct form of each answer.

45. $\tan^2 x - \tan^2 x \sin^2 x$
46. $\sin^2 x \csc^2 x - \sin^2 x$
47. $\sin^2 x \sec^2 x - \sin^2 x$
48. $\cos^2 x + \cos^2 x \tan^2 x$
49. $\dfrac{\sec^2 x - 1}{\sec x - 1}$
50. $\dfrac{\cos^2 x - 4}{\cos x - 2}$
51. $\tan^4 x + 2\tan^2 x + 1$
52. $1 - 2\cos^2 x + \cos^4 x$
53. $\sin^4 x - \cos^4 x$
54. $\sec^4 x - \tan^4 x$
55. $\csc^3 x - \csc^2 x - \csc x + 1$
56. $\sec^3 x - \sec^2 x - \sec x + 1$

In Exercises 57–60, perform the multiplication and use the fundamental identities to simplify. There is more than one correct form of each answer.

57. $(\sin x + \cos x)^2$
58. $(\cot x + \csc x)(\cot x - \csc x)$
59. $(2\csc x + 2)(2\csc x - 2)$
60. $(3 - 3\sin x)(3 + 3\sin x)$

In Exercises 61–64, perform the addition or subtraction and use the fundamental identities to simplify. There is more than one correct form of each answer.

61. $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x}$
62. $\dfrac{1}{\sec x + 1} - \dfrac{1}{\sec x - 1}$
63. $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x}$
64. $\tan x - \dfrac{\sec^2 x}{\tan x}$

In Exercises 65–68, rewrite the expression so that it is not in fractional form. There is more than one correct form of each answer.

65. $\dfrac{\sin^2 y}{1 - \cos y}$
66. $\dfrac{5}{\tan x + \sec x}$
67. $\dfrac{3}{\sec x - \tan x}$
68. $\dfrac{\tan^2 x}{\csc x + 1}$

Numerical and Graphical Analysis **In Exercises 69–72, use a graphing utility to complete the table and graph the functions. Make a conjecture about y_1 and y_2.**

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1							
y_2							

69. $y_1 = \cos\left(\dfrac{\pi}{2} - x\right), \quad y_2 = \sin x$
70. $y_1 = \sec x - \cos x, \quad y_2 = \sin x \tan x$
71. $y_1 = \dfrac{\cos x}{1 - \sin x}, \quad y_2 = \dfrac{1 + \sin x}{\cos x}$
72. $y_1 = \sec^4 x - \sec^2 x, \quad y_2 = \tan^2 x + \tan^4 x$

In Exercises 73–76, use a graphing utility to determine which of the six trigonometric functions is equal to the expression. Verify your answer algebraically.

73. $\cos x \cot x + \sin x$
74. $\sec x \csc x - \tan x$
75. $\dfrac{1}{\sin x}\left(\dfrac{1}{\cos x} - \cos x\right)$
76. $\dfrac{1}{2}\left(\dfrac{1 + \sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1 + \sin\theta}\right)$

In Exercises 77–82, use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

77. $\sqrt{9 - x^2}, \quad x = 3\cos\theta$
78. $\sqrt{64 - 16x^2}, \quad x = 2\cos\theta$
79. $\sqrt{x^2 - 9}, \quad x = 3\sec\theta$
80. $\sqrt{x^2 - 4}, \quad x = 2\sec\theta$
81. $\sqrt{x^2 + 25}, \quad x = 5\tan\theta$
82. $\sqrt{x^2 + 100}, \quad x = 10\tan\theta$

In Exercises 83–86, use the trigonometric substitution to write the algebraic equation as a trigonometric function of θ, where $-\pi/2 < \theta < \pi/2$. Then find $\sin\theta$ and $\cos\theta$.

83. $3 = \sqrt{9 - x^2}, \quad x = 3\sin\theta$
84. $3 = \sqrt{36 - x^2}, \quad x = 6\sin\theta$
85. $2\sqrt{2} = \sqrt{16 - 4x^2}, \quad x = 2\cos\theta$
86. $-5\sqrt{3} = \sqrt{100 - x^2}, \quad x = 10\cos\theta$

In Exercises 87–90, use a graphing utility to solve the equation for θ, where $0 \le \theta < 2\pi$.

87. $\sin\theta = \sqrt{1 - \cos^2\theta}$
88. $\cos\theta = -\sqrt{1 - \sin^2\theta}$
89. $\sec\theta = \sqrt{1 + \tan^2\theta}$
90. $\csc\theta = \sqrt{1 + \cot^2\theta}$

In Exercises 91–94, rewrite the expression as a single logarithm and simplify the result.

91. $\ln|\cos x| - \ln|\sin x|$
92. $\ln|\sec x| + \ln|\sin x|$
93. $\ln|\cot t| + \ln(1 + \tan^2 t)$
94. $\ln(\cos^2 t) + \ln(1 + \tan^2 t)$

5.2 Exercises

VOCABULARY CHECK:

In Exercises 1 and 2, fill in the blanks.

1. An equation that is true for all real values in its domain is called an ________.
2. An equation that is true for only some values in its domain is called a ________ ________.

In Exercises 3–8, fill in the blank to complete the trigonometric identity.

3. $\dfrac{1}{\cot u} =$ ________
4. $\dfrac{\cos u}{\sin u} =$ ________
5. $\sin^2 u +$ ________ $= 1$
6. $\cos\left(\dfrac{\pi}{2} - u\right) =$ ________
7. $\csc(-u) =$ ________
8. $\sec(-u) =$ ________

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–38, verify the identity.

1. $\sin t \csc t = 1$
2. $\sec y \cos y = 1$
3. $(1 + \sin \alpha)(1 - \sin \alpha) = \cos^2 \alpha$
4. $\cot^2 y(\sec^2 y - 1) = 1$
5. $\cos^2 \beta - \sin^2 \beta = 1 - 2\sin^2 \beta$
6. $\cos^2 \beta - \sin^2 \beta = 2\cos^2 \beta - 1$
7. $\sin^2 \alpha - \sin^4 \alpha = \cos^2 \alpha - \cos^4 \alpha$
8. $\cos x + \sin x \tan x = \sec x$
9. $\dfrac{\csc^2 \theta}{\cot \theta} = \csc \theta \sec \theta$
10. $\dfrac{\cot^3 t}{\csc t} = \cos t(\csc^2 t - 1)$
11. $\dfrac{\cot^2 t}{\csc t} = \csc t - \sin t$
12. $\dfrac{1}{\tan \beta} + \tan \beta = \dfrac{\sec^2 \beta}{\tan \beta}$
13. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \cos^3 x\sqrt{\sin x}$
14. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^5 x \tan^3 x$
15. $\dfrac{1}{\sec x \tan x} = \csc x - \sin x$
16. $\dfrac{\sec \theta - 1}{1 - \cos \theta} = \sec \theta$
17. $\csc x - \sin x = \cos x \cot x$
18. $\sec x - \cos x = \sin x \tan x$
19. $\dfrac{1}{\tan x} + \dfrac{1}{\cot x} = \tan x + \cot x$
20. $\dfrac{1}{\sin x} - \dfrac{1}{\csc x} = \csc x - \sin x$
21. $\dfrac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \csc \theta$
22. $\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta} = 2\sec \theta$
23. $\dfrac{1}{\sin x + 1} + \dfrac{1}{\csc x + 1} = 1$
24. $\cos x - \dfrac{\cos x}{1 - \tan x} = \dfrac{\sin x \cos x}{\sin x - \cos x}$
25. $\tan\left(\dfrac{\pi}{2} - \theta\right)\tan \theta = 1$
26. $\dfrac{\cos[(\pi/2) - x]}{\sin[(\pi/2) - x]} = \tan x$
27. $\dfrac{\csc(-x)}{\sec(-x)} = -\cot x$
28. $(1 + \sin y)[1 + \sin(-y)] = \cos^2 y$
29. $\dfrac{\tan x \cot x}{\cos x} = \sec x$
30. $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\cot x + \cot y}{\cot x \cot y - 1}$
31. $\dfrac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$
32. $\dfrac{\cos x - \cos y}{\sin x + \sin y} + \dfrac{\sin x - \sin y}{\cos x + \cos y} = 0$
33. $\sqrt{\dfrac{1 + \sin \theta}{1 - \sin \theta}} = \dfrac{1 + \sin \theta}{|\cos \theta|}$
34. $\sqrt{\dfrac{1 - \cos \theta}{1 + \cos \theta}} = \dfrac{1 - \cos \theta}{|\sin \theta|}$
35. $\cos^2 \beta + \cos^2\left(\dfrac{\pi}{2} - \beta\right) = 1$
36. $\sec^2 y - \cot^2\left(\dfrac{\pi}{2} - y\right) = 1$
37. $\sin t \csc\left(\dfrac{\pi}{2} - t\right) = \tan t$
38. $\sec^2\left(\dfrac{\pi}{2} - x\right) - 1 = \cot^2 x$

5.3 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. The equation $2 \sin \theta + 1 = 0$ has the solutions $\theta = \frac{7\pi}{6} + 2n\pi$ and $\theta = \frac{11\pi}{6} + 2n\pi$, which are called ________ solutions.
2. The equation $2 \tan^2 x - 3 \tan x + 1 = 0$ is a trigonometric equation that is of ________ type.
3. A solution to an equation that does not satisfy the original equation is called an ________ solution.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–6, verify that the x-values are solutions of the equation.

1. $2 \cos x - 1 = 0$
   (a) $x = \frac{\pi}{3}$ (b) $x = \frac{5\pi}{3}$
2. $\sec x - 2 = 0$
   (a) $x = \frac{\pi}{3}$ (b) $x = \frac{5\pi}{3}$
3. $3 \tan^2 2x - 1 = 0$
   (a) $x = \frac{\pi}{12}$ (b) $x = \frac{5\pi}{12}$
4. $2 \cos^2 4x - 1 = 0$
   (a) $x = \frac{\pi}{16}$ (b) $x = \frac{3\pi}{16}$
5. $2 \sin^2 x - \sin x - 1 = 0$
   (a) $x = \frac{\pi}{2}$ (b) $x = \frac{7\pi}{6}$
6. $\csc^4 x - 4 \csc^2 x = 0$
   (a) $x = \frac{\pi}{6}$ (b) $x = \frac{5\pi}{6}$

In Exercises 7–20, solve the equation.

7. $2 \cos x + 1 = 0$
8. $2 \sin x + 1 = 0$
9. $\sqrt{3} \csc x - 2 = 0$
10. $\tan x + \sqrt{3} = 0$
11. $3 \sec^2 x - 4 = 0$
12. $3 \cot^2 x - 1 = 0$
13. $\sin x(\sin x + 1) = 0$
14. $(3 \tan^2 x - 1)(\tan^2 x - 3) = 0$
15. $4 \cos^2 x - 1 = 0$
16. $\sin^2 x = 3 \cos^2 x$
17. $2 \sin^2 2x = 1$
18. $\tan^2 3x = 3$
19. $\tan 3x(\tan x - 1) = 0$
20. $\cos 2x(2 \cos x + 1) = 0$

In Exercises 21–34, find all solutions of the equation in the interval $[0, 2\pi)$.

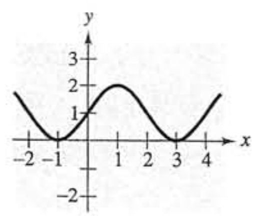
21. $\cos^3 x = \cos x$
22. $\sec^2 x - 1 = 0$
23. $3 \tan^3 x = \tan x$
24. $2 \sin^2 x = 2 + \cos x$
25. $\sec^2 x - \sec x = 2$
26. $\sec x \csc x = 2 \csc x$
27. $2 \sin x + \csc x = 0$
28. $\sec x + \tan x = 1$
29. $2 \cos^2 x + \cos x - 1 = 0$
30. $2 \sin^2 x + 3 \sin x + 1 = 0$
31. $2 \sec^2 x + \tan^2 x - 3 = 0$
32. $\cos x + \sin x \tan x = 2$
33. $\csc x + \cot x = 1$
34. $\sin x - 2 = \cos x - 2$

In Exercises 35–40, solve the multiple-angle equation.

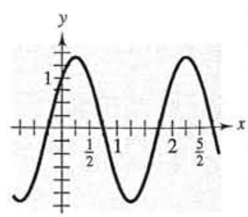
35. $\cos 2x = \frac{1}{2}$
36. $\sin 2x = -\frac{\sqrt{3}}{2}$
37. $\tan 3x = 1$
38. $\sec 4x = 2$
39. $\cos \frac{x}{2} = \frac{\sqrt{2}}{2}$
40. $\sin \frac{x}{2} = -\frac{\sqrt{3}}{2}$

In Exercises 41–44, find the x-intercepts of the graph.

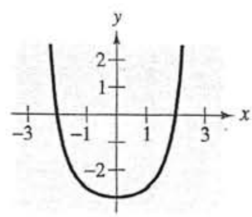
41. $y = \sin \frac{\pi x}{2} + 1$



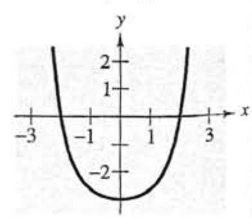
42. $y = \sin \pi x + \cos \pi x$



43. $y = \tan^2\left(\frac{\pi x}{6}\right) - 3$



44. $y = \sec^4\left(\frac{\pi x}{8}\right) - 4$



5 Review Exercises

5.1 In Exercises 1–6, name the trigonometric function that is equivalent to the expression.

1. $\dfrac{1}{\cos x}$
2. $\dfrac{1}{\sin x}$
3. $\dfrac{1}{\sec x}$
4. $\dfrac{1}{\tan x}$
5. $\dfrac{\cos x}{\sin x}$
6. $\sqrt{1 + \tan^2 x}$

In Exercises 7–10, use the given values and trigonometric identities to evaluate (if possible) all six trigonometric functions.

7. $\sin x = \frac{3}{5}$, $\cos x = \frac{4}{5}$
8. $\tan \theta = \dfrac{2}{3}$, $\sec \theta = \dfrac{\sqrt{13}}{3}$
9. $\sin\left(\dfrac{\pi}{2} - x\right) = \dfrac{\sqrt{2}}{2}$, $\sin x = -\dfrac{\sqrt{2}}{2}$
10. $\csc\left(\dfrac{\pi}{2} - \theta\right) = 9$, $\sin \theta = \dfrac{4\sqrt{5}}{9}$

In Exercises 11–22, use the fundamental trigonometric identities to simplify the expression.

11. $\dfrac{1}{\cot^2 x + 1}$
12. $\dfrac{\tan \theta}{1 - \cos^2 \theta}$
13. $\tan^2 x(\csc^2 x - 1)$
14. $\cot^2 x(\sin^2 x)$
15. $\dfrac{\sin\left(\dfrac{\pi}{2} - \theta\right)}{\sin \theta}$
16. $\dfrac{\cot\left(\dfrac{\pi}{2} - u\right)}{\cos u}$
17. $\cos^2 x + \cos^2 x \cot^2 x$
18. $\tan^2 \theta \csc^2 \theta - \tan^2 \theta$
19. $(\tan x + 1)^2 \cos x$
20. $(\sec x - \tan x)^2$
21. $\dfrac{1}{\csc \theta + 1} - \dfrac{1}{\csc \theta - 1}$
22. $\dfrac{\cos^2 x}{1 - \sin x}$

23. ***Rate of Change*** The rate of change of the function $f(x) = \csc x - \cot x$ is given by the expression $\csc^2 x - \csc x \cot x$. Show that this expression can also be written as

$$\frac{1 - \cos x}{\sin^2 x}.$$

24. ***Rate of Change*** The rate of change of the function $f(x) = 2\sqrt{\sin x}$ is given by the expression $\sin^{-1/2} x \cos x$. Show that this expression can also be written as $\cot x\sqrt{\sin x}$.

5.2 In Exercises 25–32, verify the identity.

25. $\cos x(\tan^2 x + 1) = \sec x$
26. $\sec^2 x \cot x - \cot x = \tan x$
27. $\cos\left(x + \dfrac{\pi}{2}\right) = -\sin x$
28. $\cot\left(\dfrac{\pi}{2} - x\right) = \tan x$
29. $\dfrac{1}{\tan \theta \csc \theta} = \cos \theta$
30. $\dfrac{1}{\tan x \csc x \sin x} = \cot x$
31. $\sin^5 x \cos^2 x = (\cos^2 x - 2\cos^4 x + \cos^6 x) \sin x$
32. $\cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x) \cos x$

5.3 In Exercises 33–38, solve the equation.

33. $\sin x = \sqrt{3} - \sin x$
34. $4 \cos \theta = 1 + 2 \cos \theta$
35. $3\sqrt{3} \tan u = 3$
36. $\frac{1}{2} \sec x - 1 = 0$
37. $3 \csc^2 x = 4$
38. $4 \tan^2 u - 1 = \tan^2 u$

In Exercises 39–46, find all solutions of the equation in the interval $[0, 2\pi)$.

39. $2 \cos^2 x - \cos x = 1$
40. $2 \sin^2 x - 3 \sin x = -1$
41. $\cos^2 x + \sin x = 1$
42. $\sin^2 x + 2 \cos x = 2$
43. $2 \sin 2x - \sqrt{2} = 0$
44. $\sqrt{3} \tan 3x = 0$
45. $\cos 4x(\cos x - 1) = 0$
46. $3 \csc^2 5x = -4$

In Exercises 47–50, use inverse functions where needed to find all solutions of the equation in the interval $[0, 2\pi)$.

47. $\sin^2 x - 2 \sin x = 0$
48. $2 \cos^2 x + 3 \cos x = 0$
49. $\tan^2 \theta + \tan \theta - 12 = 0$
50. $\sec^2 x + 6 \tan x + 4 = 0$

5.4 In Exercises 51–54, find the exact values of the sine, cosine, and tangent of the angle by using a sum or difference formula.

51. $285° = 315° - 30°$
52. $345° = 300° + 45°$
53. $\dfrac{25\pi}{12} = \dfrac{11\pi}{6} + \dfrac{\pi}{4}$
54. $\dfrac{19\pi}{12} = \dfrac{11\pi}{6} - \dfrac{\pi}{4}$

5.4 Exercises

VOCABULARY CHECK: Fill in the blank to complete the trigonometric identity.

1. $\sin(u - v) =$ ________ **2.** $\cos(u + v) =$ ________

3. $\tan(u + v) =$ ________ **4.** $\sin(u + v) =$ ________

5. $\cos(u - v) =$ ________ **6.** $\tan(u - v) =$ ________

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–6, find the exact value of each expression.

1. (a) $\cos(120° + 45°)$ (b) $\cos 120° + \cos 45°$

2. (a) $\sin(135° - 30°)$ (b) $\sin 135° - \cos 30°$

3. (a) $\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$ (b) $\cos\frac{\pi}{4} + \cos\frac{\pi}{3}$

4. (a) $\sin\left(\frac{3\pi}{4} + \frac{5\pi}{6}\right)$ (b) $\sin\frac{3\pi}{4} + \sin\frac{5\pi}{6}$

5. (a) $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right)$ (b) $\sin\frac{7\pi}{6} - \sin\frac{\pi}{3}$

6. (a) $\sin(315° - 60°)$ (b) $\sin 315° - \sin 60°$

In Exercises 7–22, find the exact values of the sine, cosine, and tangent of the angle by using a sum or difference formula.

7. $105° = 60° + 45°$ **8.** $165° = 135° + 30°$

9. $195° = 225° - 30°$ **10.** $255° = 300° - 45°$

11. $\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$ **12.** $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$

13. $\frac{17\pi}{12} = \frac{9\pi}{4} - \frac{5\pi}{6}$ **14.** $-\frac{\pi}{12} = \frac{\pi}{6} - \frac{\pi}{4}$

15. $285°$ **16.** $-105°$

17. $-165°$ **18.** $15°$

19. $\frac{13\pi}{12}$ **20.** $-\frac{7\pi}{12}$

21. $-\frac{13\pi}{12}$ **22.** $\frac{5\pi}{12}$

In Exercises 23–30, write the expression as the sine, cosine, or tangent of an angle.

23. $\cos 25° \cos 15° - \sin 25° \sin 15°$

24. $\sin 140° \cos 50° + \cos 140° \sin 50°$

25. $\frac{\tan 325° - \tan 86°}{1 + \tan 325° \tan 86°}$

26. $\frac{\tan 140° - \tan 60°}{1 + \tan 140° \tan 60°}$

27. $\sin 3 \cos 1.2 - \cos 3 \sin 1.2$

28. $\cos\frac{\pi}{7}\cos\frac{\pi}{5} - \sin\frac{\pi}{7}\sin\frac{\pi}{5}$

29. $\frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

30. $\cos 3x \cos 2y + \sin 3x \sin 2y$

In Exercises 31–36, find the exact value of the expression.

31. $\sin 330° \cos 30° - \cos 330° \sin 30°$

32. $\cos 15° \cos 60° + \sin 15° \sin 60°$

33. $\sin\frac{\pi}{12}\cos\frac{\pi}{4} + \cos\frac{\pi}{12}\sin\frac{\pi}{4}$

34. $\cos\frac{\pi}{16}\cos\frac{3\pi}{16} - \sin\frac{\pi}{16}\sin\frac{3\pi}{16}$

35. $\frac{\tan 25° + \tan 110°}{1 - \tan 25° \tan 110°}$

36. $\frac{\tan(5\pi/4) - \tan(\pi/12)}{1 + \tan(5\pi/4)\tan(\pi/12)}$

In Exercises 37–44, find the exact value of the trigonometric function given that $\sin u = \frac{5}{13}$ and $\cos v = -\frac{3}{5}$. (Both u and v are in Quadrant II.)

37. $\sin(u + v)$ **38.** $\cos(u - v)$

39. $\cos(u + v)$ **40.** $\sin(v - u)$

41. $\tan(u + v)$ **42.** $\csc(u - v)$

43. $\sec(v - u)$ **44.** $\cot(u + v)$

In Exercises 45–50, find the exact value of the trigonometric function given that $\sin u = -\frac{7}{25}$ and $\cos v = -\frac{4}{5}$. (Both u and v are in Quadrant III.)

45. $\cos(u + v)$ **46.** $\sin(u + v)$

47. $\tan(u - v)$ **48.** $\cot(v - u)$

49. $\sec(u + v)$ **50.** $\cos(u - v)$

In Exercises 51–54, write the trigonometric expression as an algebraic expression.

51. $\sin(\arcsin x + \arccos x)$
52. $\sin(\arctan 2x - \arccos x)$
53. $\cos(\arccos x + \arcsin x)$
54. $\cos(\arccos x - \arctan x)$

In Exercises 55–64, verify the identity.

55. $\sin(3\pi - x) = \sin x$
56. $\sin\left(\dfrac{\pi}{2} + x\right) = \cos x$
57. $\sin\left(\dfrac{\pi}{6} + x\right) = \dfrac{1}{2}(\cos x + \sqrt{3}\sin x)$
58. $\cos\left(\dfrac{5\pi}{4} - x\right) = -\dfrac{\sqrt{2}}{2}(\cos x + \sin x)$
59. $\cos(\pi - \theta) + \sin\left(\dfrac{\pi}{2} + \theta\right) = 0$
60. $\tan\left(\dfrac{\pi}{4} - \theta\right) = \dfrac{1 - \tan\theta}{1 + \tan\theta}$
61. $\cos(x + y)\cos(x - y) = \cos^2 x - \sin^2 y$
62. $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$
63. $\sin(x + y) + \sin(x - y) = 2\sin x\cos y$
64. $\cos(x + y) + \cos(x - y) = 2\cos x\cos y$

In Exercises 65–68, simplify the expression algebraically and use a graphing utility to confirm your answer graphically.

65. $\cos\left(\dfrac{3\pi}{2} - x\right)$
66. $\cos(\pi + x)$
67. $\sin\left(\dfrac{3\pi}{2} + \theta\right)$
68. $\tan(\pi + \theta)$

In Exercises 69–72, find all solutions of the equation in the interval $[0, 2\pi)$.

69. $\sin\left(x + \dfrac{\pi}{3}\right) + \sin\left(x - \dfrac{\pi}{3}\right) = 1$
70. $\sin\left(x + \dfrac{\pi}{6}\right) - \sin\left(x - \dfrac{\pi}{6}\right) = \dfrac{1}{2}$
71. $\cos\left(x + \dfrac{\pi}{4}\right) - \cos\left(x - \dfrac{\pi}{4}\right) = 1$
72. $\tan(x + \pi) + 2\sin(x + \pi) = 0$

In Exercises 73 and 74, use a graphing utility to approximate the solutions in the interval $[0, 2\pi)$.

73. $\cos\left(x + \dfrac{\pi}{4}\right) + \cos\left(x - \dfrac{\pi}{4}\right) = 1$
74. $\tan(x + \pi) - \cos\left(x + \dfrac{\pi}{2}\right) = 0$

Model It

75. ***Harmonic Motion*** A weight is attached to a spring suspended vertically from a ceiling. When a driving force is applied to the system, the weight moves vertically from its equilibrium position, and this motion is modeled by

$$y = \frac{1}{3}\sin 2t + \frac{1}{4}\cos 2t$$

where y is the distance from equilibrium (in feet) and t is the time (in seconds).

(a) Use the identity

$$a\sin B\theta + b\cos B\theta = \sqrt{a^2 + b^2}\sin(B\theta + C)$$

where $C = \arctan(b/a)$, $a > 0$, to write the model in the form

$$y = \sqrt{a^2 + b^2}\sin(Bt + C).$$

(b) Find the amplitude of the oscillations of the weight.

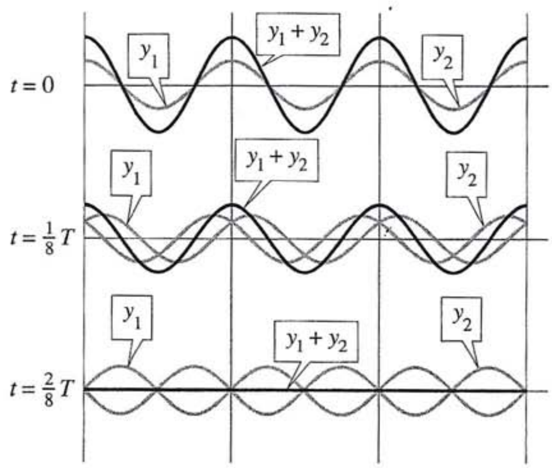
(c) Find the frequency of the oscillations of the weight.

76. ***Standing Waves*** The equation of a standing wave is obtained by adding the displacements of two waves traveling in opposite directions (see figure). Assume that each of the waves has amplitude A, period T, and wavelength λ. If the models for these waves are

$$y_1 = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{and} \quad y_2 = A\cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

show that

$$y_1 + y_2 = 2A\cos\frac{2\pi t}{T}\cos\frac{2\pi x}{\lambda}.$$



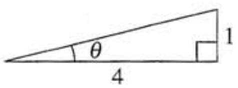
5.5 Exercises

VOCABULARY CHECK: Fill in the blank to complete the trigonometric formula.

1. $\sin 2u =$ ________
2. $\dfrac{1 + \cos 2u}{2} =$ ________
3. $\cos 2u =$ ________
4. $\dfrac{1 - \cos 2u}{1 + \cos 2u} =$ ________
5. $\sin \dfrac{u}{2} =$ ________
6. $\tan \dfrac{u}{2} =$ ________
7. $\cos u \cos v =$ ________
8. $\sin u \cos v =$ ________
9. $\sin u + \sin v =$ ________
10. $\cos u - \cos v =$ ________

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–8, use the figure to find the exact value of the trigonometric function.



1. $\sin \theta$
2. $\tan \theta$
3. $\cos 2\theta$
4. $\sin 2\theta$
5. $\tan 2\theta$
6. $\sec 2\theta$
7. $\csc 2\theta$
8. $\cot 2\theta$

In Exercises 9–18, find the exact solutions of the equation in the interval $[0, 2\pi)$.

9. $\sin 2x - \sin x = 0$
10. $\sin 2x + \cos x = 0$
11. $4 \sin x \cos x = 1$
12. $\sin 2x \sin x = \cos x$
13. $\cos 2x - \cos x = 0$
14. $\cos 2x + \sin x = 0$
15. $\tan 2x - \cot x = 0$
16. $\tan 2x - 2 \cos x = 0$
17. $\sin 4x = -2 \sin 2x$
18. $(\sin 2x + \cos 2x)^2 = 1$

In Exercises 19–22, use a double-angle formula to rewrite the expression.

19. $6 \sin x \cos x$
20. $6 \cos^2 x - 3$
21. $4 - 8 \sin^2 x$
22. $(\cos x + \sin x)(\cos x - \sin x)$

In Exercises 23–28, find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

23. $\sin u = -\dfrac{4}{5}, \quad \pi < u < \dfrac{3\pi}{2}$

24. $\cos u = -\dfrac{2}{3}, \quad \dfrac{\pi}{2} < u < \pi$

25. $\tan u = \dfrac{3}{4}, \quad 0 < u < \dfrac{\pi}{2}$

26. $\cot u = -4, \quad \dfrac{3\pi}{2} < u < 2\pi$

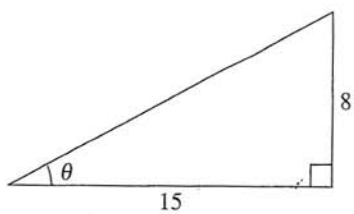
27. $\sec u = -\dfrac{5}{2}, \quad \dfrac{\pi}{2} < u < \pi$

28. $\csc u = 3, \quad \dfrac{\pi}{2} < u < \pi$

In Exercises 29–34, use the power-reducing formulas to rewrite the expression in terms of the first power of the cosine.

29. $\cos^4 x$
30. $\sin^8 x$
31. $\sin^2 x \cos^2 x$
32. $\sin^4 x \cos^4 x$
33. $\sin^2 x \cos^4 x$
34. $\sin^4 x \cos^2 x$

In Exercises 35–40, use the figure to find the exact value of the trigonometric function.



35. $\cos \dfrac{\theta}{2}$
36. $\sin \dfrac{\theta}{2}$
37. $\tan \dfrac{\theta}{2}$
38. $\sec \dfrac{\theta}{2}$
39. $\csc \dfrac{\theta}{2}$
40. $\cot \dfrac{\theta}{2}$

In Exercises 41–48, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

41. $75°$ **42.** $165°$

43. $112° 30'$ **44.** $67° 30'$

45. $\frac{\pi}{8}$ **46.** $\frac{\pi}{12}$

47. $\frac{3\pi}{8}$ **48.** $\frac{7\pi}{12}$

In Exercises 49–54, find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ using the half-angle formulas.

49. $\sin u = \frac{5}{13}, \quad \frac{\pi}{2} < u < \pi$

50. $\cos u = \frac{3}{5}, \quad 0 < u < \frac{\pi}{2}$

51. $\tan u = -\frac{5}{8}, \quad \frac{3\pi}{2} < u < 2\pi$

52. $\cot u = 3, \quad \pi < u < \frac{3\pi}{2}$

53. $\csc u = -\frac{5}{3}, \quad \pi < u < \frac{3\pi}{2}$

54. $\sec u = -\frac{7}{2}, \quad \frac{\pi}{2} < u < \pi$

In Exercises 55–58, use the half-angle formulas to simplify the expression.

55. $\sqrt{\frac{1 - \cos 6x}{2}}$ **56.** $\sqrt{\frac{1 + \cos 4x}{2}}$

57. $-\sqrt{\frac{1 - \cos 8x}{1 + \cos 8x}}$ **58.** $-\sqrt{\frac{1 - \cos(x - 1)}{2}}$

In Exercises 59–62, find all solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to graph the equation and verify the solutions.

59. $\sin\frac{x}{2} + \cos x = 0$ **60.** $\sin\frac{x}{2} + \cos x - 1 = 0$

61. $\cos\frac{x}{2} - \sin x = 0$ **62.** $\tan\frac{x}{2} - \sin x = 0$

In Exercises 63–74, use the product-to-sum formulas to write the product as a sum or difference.

63. $6 \sin\frac{\pi}{4} \cos\frac{\pi}{4}$ **64.** $4 \cos\frac{\pi}{3} \sin\frac{5\pi}{6}$

65. $10 \cos 75° \cos 15°$ **66.** $6 \sin 45° \cos 15°$

67. $\cos 4\theta \sin 6\theta$ **68.** $3 \sin 2\alpha \sin 3\alpha$

69. $5 \cos(-5\beta) \cos 3\beta$ **70.** $\cos 2\theta \cos 4\theta$

71. $\sin(x + y) \sin(x - y)$ **72.** $\sin(x + y) \cos(x - y)$

73. $\cos(\theta - \pi) \sin(\theta + \pi)$ **74.** $\sin(\theta + \pi) \sin(\theta - \pi)$

In Exercises 75–82, use the sum-to-product formulas to write the sum or difference as a product.

75. $\sin 5\theta - \sin 3\theta$ **76.** $\sin 3\theta + \sin \theta$

77. $\cos 6x + \cos 2x$ **78.** $\sin x + \sin 5x$

79. $\sin(\alpha + \beta) - \sin(\alpha - \beta)$

80. $\cos(\phi + 2\pi) + \cos \phi$

81. $\cos\left(\theta + \frac{\pi}{2}\right) - \cos\left(\theta - \frac{\pi}{2}\right)$

82. $\sin\left(x + \frac{\pi}{2}\right) + \sin\left(x - \frac{\pi}{2}\right)$

In Exercises 83–86, use the sum-to-product formulas to find the exact value of the expression.

83. $\sin 60° + \sin 30°$ **84.** $\cos 120° + \cos 30°$

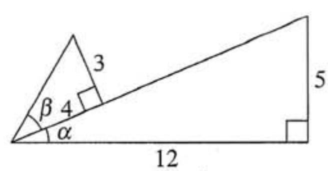
85. $\cos\frac{3\pi}{4} - \cos\frac{\pi}{4}$ **86.** $\sin\frac{5\pi}{4} - \sin\frac{3\pi}{4}$

In Exercises 87–90, find all solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to graph the equation and verify the solutions.

87. $\sin 6x + \sin 2x = 0$ **88.** $\cos 2x - \cos 6x = 0$

89. $\frac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$ **90.** $\sin^2 3x - \sin^2 x = 0$

In Exercises 91–94, use the figure and trigonometric identities to find the exact value of the trigonometric function in two ways.



91. $\sin^2 \alpha$ **92.** $\cos^2 \alpha$

93. $\sin \alpha \cos \beta$ **94.** $\cos \alpha \sin \beta$

In Exercises 95–110, verify the identity.

95. $\csc 2\theta = \frac{\csc \theta}{2 \cos \theta}$

96. $\sec 2\theta = \frac{\sec^2 \theta}{2 - \sec^2 \theta}$

97. $\cos^2 2\alpha - \sin^2 2\alpha = \cos 4\alpha$

98. $\cos^4 x - \sin^4 x = \cos 2x$

99. $(\sin x + \cos x)^2 = 1 + \sin 2x$